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The electric field due to a small "chunk" $\Delta q$ of charge is

$$\Delta \vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{\Delta q \hat{r}}{r^2}$$

unit vector from $\Delta q$ to wherever you want to calculate $\Delta \vec{E}$

The electric field due to collection of "chunks" of charge is

$$\vec{E} = \sum_i \Delta \vec{E}_i = \frac{1}{4\pi \varepsilon_0} \sum_i \frac{\Delta q_i \hat{r}_i}{r_i^2}$$

unit vector from $\Delta q_i$ to wherever you want to calculate $\vec{E}$

As $\Delta q \rightarrow dq \rightarrow 0$, the sum becomes an integral.
If charge is distributed along a straight line segment parallel to the x-axis, the amount of charge $dq$ on a segment of length $dx$ is $\lambda \, dx$.

$\lambda$ is the linear density of charge (amount of charge per unit length). $\lambda$ may be a function of position.

Think $\lambda \Leftrightarrow \ell \Leftrightarrow$ length. $\lambda$ times the length of line segment is the total charge on the line segment.
The electric field at point \( P \) due to the charge \( dq \) is

\[
d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r'^2} \hat{r}' = \frac{1}{4\pi\varepsilon_0} \frac{\lambda dx}{r'^2} \hat{r}'
\]

I'm assuming positively charged objects in these “distribution of charges” slides.

I would start a homework or test problem with this:

\[
dE = k \frac{|dq|}{r^2}
\]
The electric field at P due to the entire line of charge is

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\hat{r}' \lambda(x) \, dx}{r'^2} . \]

The integration is carried out over the entire length of the line, which need not be straight. Also, \( \lambda \) could be a function of position, and can be taken outside the integral only if the charge distribution is uniform.
If charge is distributed over a two-dimensional surface, the amount of charge $dq$ on an infinitesimal piece of the surface is $\sigma dS$, where $\sigma$ is the surface density of charge (amount of charge per unit area).
The electric field at P due to the charge dq is

\[
\mathbf{dE} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r'^2} \hat{r}' = \frac{1}{4\pi\varepsilon_0} \frac{\sigma}{r'^2} \hat{r}'
\]
The net electric field at P due to the entire surface of charge is

\[
\vec{E} = \frac{1}{4\pi \varepsilon_0} \int_S \hat{r}' \frac{\sigma(x, y) \, dS}{r'^2}
\]
After you have seen the above, I hope you believe that the net electric field at $P$ due to a three-dimensional distribution of charge is...

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(x, y, z)}{r'^2} \, dV.$$
Summarizing:

Charge distributed along a line: \[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \hat{r}' \frac{\lambda \, dx}{r'^2}.
\]

Charge distributed over a surface: \[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_S \hat{r}' \frac{\sigma \, dS}{r'^2}.
\]

Charge distributed inside a volume: \[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_V \hat{r}' \frac{\rho \, dV}{r'^2}.
\]

If the charge distribution is uniform, then \(\lambda\), \(\sigma\), and \(\rho\) can be taken outside the integrals.
The Electric Field
Due to a Continuous Charge Distribution
(worked examples)
Example: A rod of length \( L \) has a uniform charge per unit length \( \lambda \) and a total charge \( Q \). Calculate the electric field at a point \( P \) along the axis of the rod at a distance \( d \) from one end.

Let’s put the origin at \( P \). The linear charge density and \( Q \) are related by

\[
\lambda = \frac{Q}{L} \quad \text{and} \quad Q = \lambda L
\]

Let’s assume \( Q \) is positive.
The electric field points away from the rod. By symmetry, the electric field on the axis of the rod has no y-component. $dE$ from the charge on an infinitesimal length $dx$ of rod is

\[
dE = k \frac{dq}{x^2} = k \frac{\lambda}{x^2} dx
\]

Note: $\vec{dE}$ is in the $-x$ direction. $dE$ is the magnitude of $\vec{dE}$. I’ve used the fact that $Q>0$ (so $dq=0$) to eliminate the absolute value signs in the starting equation.
\[ \overrightarrow{E} = \int_{d}^{d+L} d\overrightarrow{E}_x = -k \int_{d}^{d+L} \frac{\lambda}{x^2} dx \hat{i} = -k\lambda \int_{d}^{d+L} \frac{dx}{x^2} \hat{i} = -k\lambda \left( -\frac{1}{x} \right)_{d}^{d+L} \hat{i} \]

\[ \overrightarrow{E} = -k\lambda \left( -\frac{1}{d+L} + \frac{1}{d} \right) \hat{i} = -k\lambda \left( \frac{-d + d + L}{d(d + L)} \right) \hat{i} = -k \frac{\lambda L}{d(d + L)} \hat{i} = -\frac{kQ}{d(d + L)} \hat{i} \]
Example: A ring of radius $a$ has a uniform charge per unit length and a total positive charge $Q$. Calculate the electric field at a point $P$ along the axis of the ring at a distance $x_0$ from its center.

By symmetry, the $y$- and $z$-components of $E$ are zero, and all points on the ring are a distance $r$ from point $P$. 
\[
dE = k \frac{dQ}{r^2}
\]

\[
dE_x = k \frac{dQ}{r^2} \cos \theta
\]

\[
r = \sqrt{x_0^2 + a^2}
\]

\[
\cos \theta = \frac{x_0}{r}
\]

For a given \(x_0\), \(r\) is a constant for points on the ring.

\[
E_x = \int_{\text{ring}} dE_x = \int_{\text{ring}} \left( k \frac{dQ}{r^2} \right) \frac{x_0}{r} = k \frac{x_0}{r^3} \int_{\text{ring}} dQ = k \frac{x_0}{r^3} Q = \frac{kx_0 Q}{\left( x_0^2 + a^2 \right)^{3/2}}
\]

Or, in general, on the ring axis \(E_{x, \text{ring}} = \frac{kxQ}{\left( x^2 + a^2 \right)^{3/2}}\).
Example: A disc of radius \( R \) has a uniform charge per unit area \( \sigma \). Calculate the electric field at a point \( P \) along the central axis of the disc at a distance \( x_0 \) from its center.

The disc is made of concentric rings. The area of a ring at a radius \( r \) is \( 2\pi r \, dr \), and the charge on each ring is \( \sigma (2\pi r \, dr) \).

We can use the equation on the previous slide for the electric field due to a ring, replace \( a \) by \( r \), and integrate from \( r = 0 \) to \( r = R \).

\[
dE_{\text{ring}} = \frac{k x_0 \sigma 2\pi r \, dr}{(x_0^2 + r^2)^{3/2}}.\]

Caution! I’ve switched the “meaning” of \( r \)!
\[
E_x = \int_{\text{disc}} dE_x = \int_{\text{disc}} \frac{k x_0 \sigma 2\pi r dr}{(x_0^2 + r^2)^{3/2}} = k x_0 \pi \sigma \int_0^R \frac{2r \, dr}{(x_0^2 + r^2)^{3/2}}
\]

\[
E_x = k x_0 \pi \sigma \left[ \frac{(x_0^2 + r^2)^{-1/2}}{R} \right]_{R_0}^{R} = 2k \pi \sigma \left( \frac{x_0}{|x_0|} - \frac{x_0}{(x_0^2 + R^2)^{1/2}} \right)
\]
Example: Calculate the electric field at a distance $x_0$ from an infinite plane sheet with a uniform charge density $\sigma$.

Treat the infinite sheet as disc of infinite radius.

Let $R \to \infty$ and use $k = \frac{1}{4\pi \varepsilon_0}$ to get

$$E_{\text{sheet}} = \frac{|\sigma|}{2\varepsilon_0}.$$ 

Interesting...does not depend on distance from the sheet.

I’ve been Really Nice and put this on your starting equation sheet. You don’t have to derive it for your homework!
Electric Field Lines

Electric field lines help us visualize the electric field and predict how charged particles would respond the field.

Example: electric field lines for isolated +2e and -e charges.
Here’s how electric field lines are related to the field:

• The electric field vector $\mathbf{E}$ is tangent to the field lines.

• The number of lines per unit area through a surface perpendicular to the lines is proportional to the electric field strength in that region.

• The field lines begin on positive charges and end on negative charges.

• The number of lines leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.

• No two field lines can cross.

Example: draw the electric field lines for charges $+2e$ and $-1e$, separated by a fixed distance. Easier to use this link!